ECE 333

GREEN ELECTRIC ENERGY

12. The Solar Energy Resource

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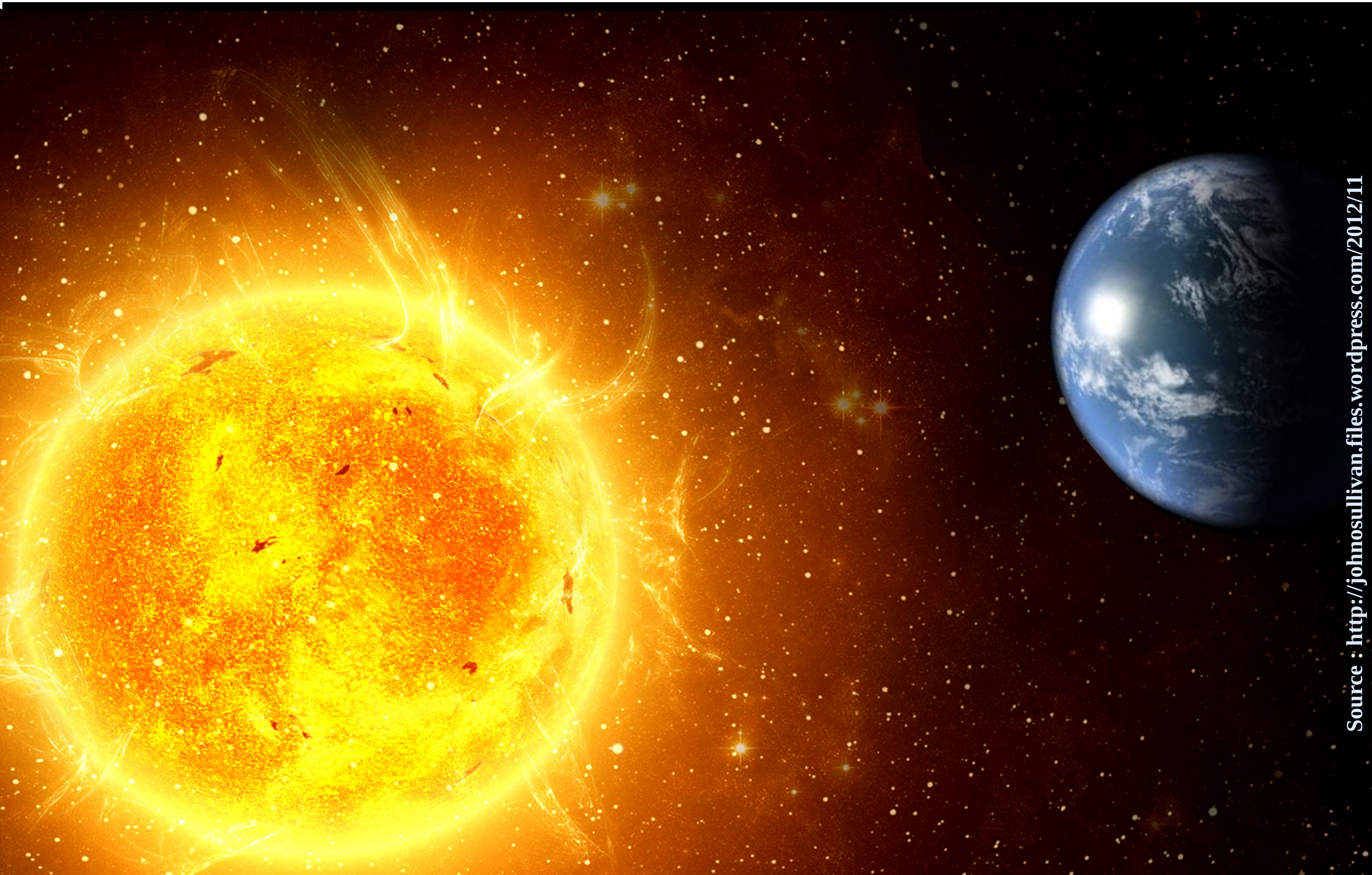
SOLAR ENERGY

- Solar energy is the most abundant renewable energy source and is very clean
- Solar energy is harnessed for various purposes, including electricity generation, lighting and steam and hot water production

SOLAR RESOURCE

- ☐ The solar energy source
- ☐ Extraterrestrial solar irradiation
- ☐ Analysis of solar position in the sky and its application to the determination of
  - ○ optimal tilt angle design for a solar panel
  - ○ sun path diagram for shading analysis
  - ○ solar time and civil time relationship

UNDERLYING BASIS: THE SUN IS A LIMITLESS ENERGY SOURCE



Source : http://johnosullivan.files.wordpress.com/2012/11

SOLAR ENERGY

- The *thermonuclear reactions* – hydrogen atoms fuse together to form helium – in the sun are the source of solar energy
- In every second, roughly 4 *billion kg* of mass are converted into energy, as described by Einstein’s well–known *mass-energy equation* $E = mc^2$
- The plentiful solar energy during the past 4 or 5 *billion years* is expected to continue in the future

SOLAR ENERGY

- ☐ This energy generated is so immense that it keeps the sun at very high temperatures
- ☐ Every object emits radiant energy in an amount that depends on its temperature; the sun emits *solar energy* into space via *radiation*
- ☐ *Insolation* or *solar irradiation* stated in units of $\frac{W}{m^2}$ measures the power density of the solar energy

PLANCK'S LAW

- ☐ In general, we use the theoretical concept of a *blackbody* – defined to be a perfect emitter, as well as a perfect absorber – to discuss radiation

- ☐ The emissive power intensity of a *blackbody* is a function of its wavelength λ and temperature τ as described by *Planck's law*

PLANCK'S LAW

emissive power intensity

$W / m^2 - \mu m$

$$\rho_\lambda(\tau) = \frac{3.74 \times 10^8}{\lambda^5 \left[exp\left(\frac{14,400}{\lambda \tau} \right) - 1 \right]}$$

μm

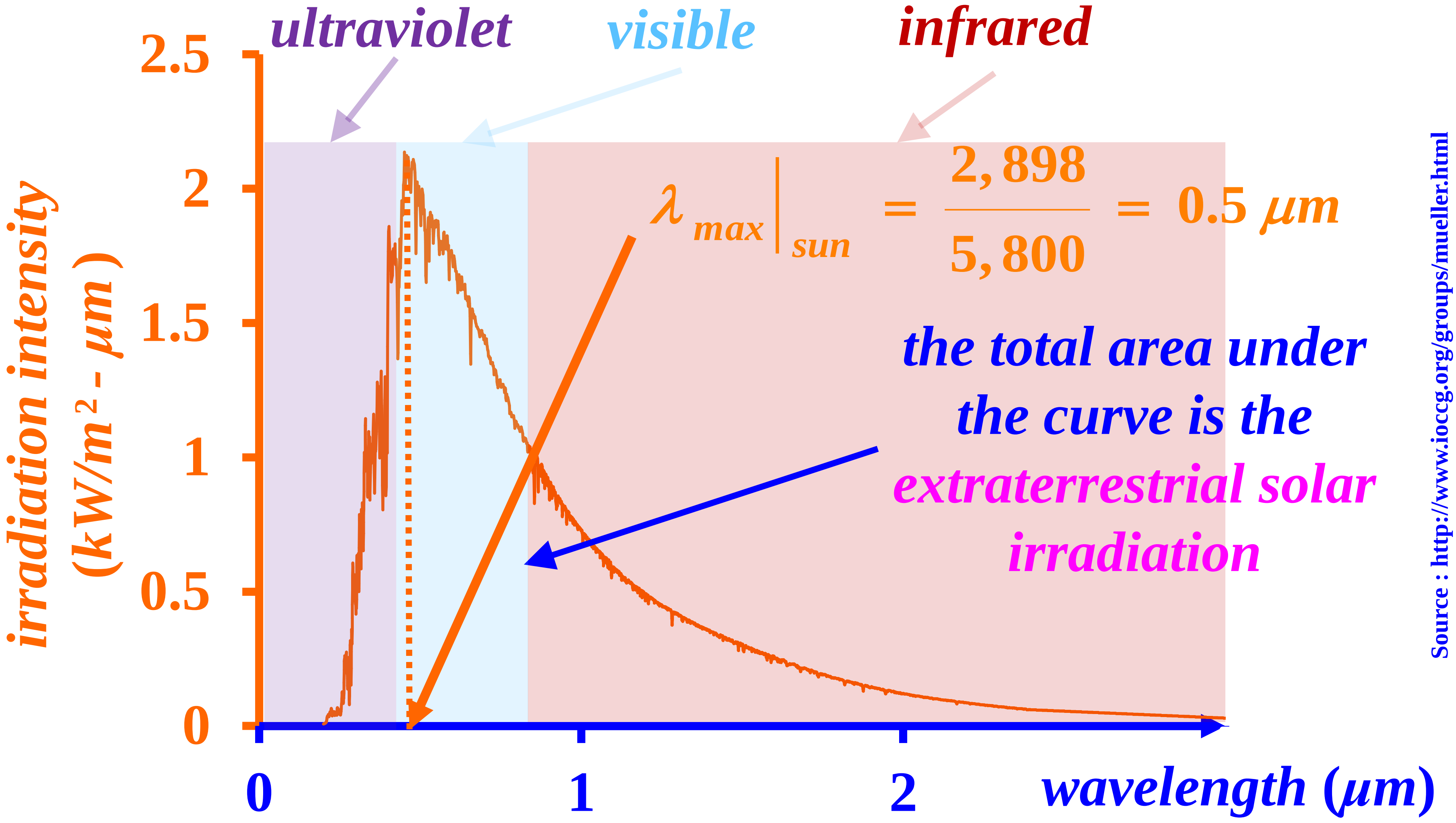
K

WIEN'S DISPLACEMENT RULE

An important feature of *blackbody* radiation is given by *Wien's displacement rule*, which determines the wavelength at which the emissive power intensity reaches its highest value

$$\lambda_{max} = \frac{2,898}{\tau} \quad \mu m$$

EXTRATERRESTRIAL SOLAR SPECTRUM



$$\lambda_{max}\Big|_{sun} = \frac{2,898}{5,800} = 0.5\ \mu m$$

the total area under the curve is the extraterrestrial solar irradiation

Source : http://www.ioccg.org/groups/mueller.html

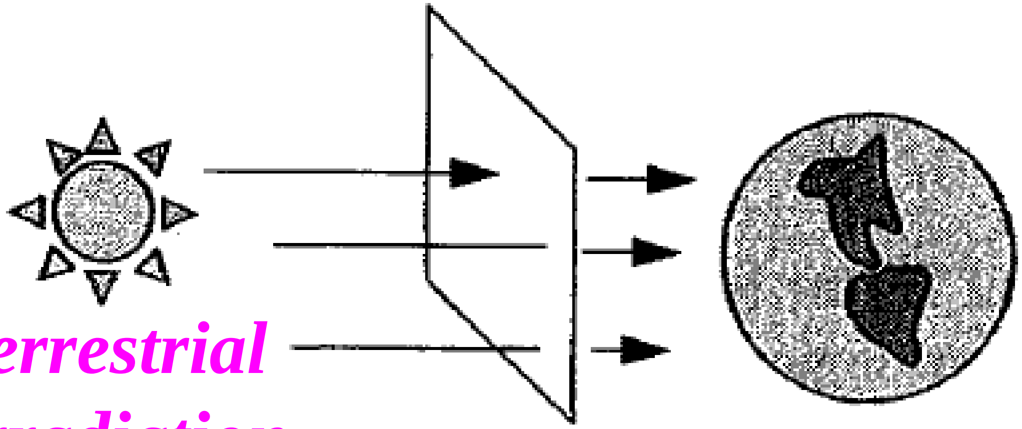
THE SOLAR IRRADIATION

- ☐ The sun's surface temperature is estimated at 5,800 K and its power density is assumed to be 1.37 kW/m^2 – the value of insolation or solar irradiation just outside the earth's atmosphere
- ☐ The sun emits maximum energy at

$$\lambda_{max}\Big|_{sun} = \frac{2,898}{5,800} = 0.5\ \mu m$$

EXTRATERRESTRIAL SOLAR IRRADIATION

Extraterrestrial solar irradiation **is defined as the solar irradiation striking an imaginary surface at the top of the earth's atmosphere, which is perpendicular to the line from the earth's center to the sun's center**



extraterrestrial solar irradiation

STEFAN–BOLTZMANN LAW OF RADIATION

- **The total area under the power intensity curve is the *blackbody* radiant power density emitted over all the wavelengths**
- **The *Stefan-Boltzmann law of radiation* states that**

the total radiant power in W

the surface area of the blackbody in m^2

$$p_{blackbody} = \sigma A \tau^4$$

Stefan-Boltzmann constant: $5.67\times10^{-8} W / m^2 \text{-} K$

THE EARTH'S RADIATION

- ☐ We consider the earth to be a blackbody with average surface temperature $15\,^{o}C$ and area equal to $5.1\times 10^{14}\ m^{2}$
- ☐ The *Stefan-Boltzmann law of radiation* states that the earth radiates

THE EARTH'S RADIATION

$$p_{earth} = \sigma A \tau^{4}$$

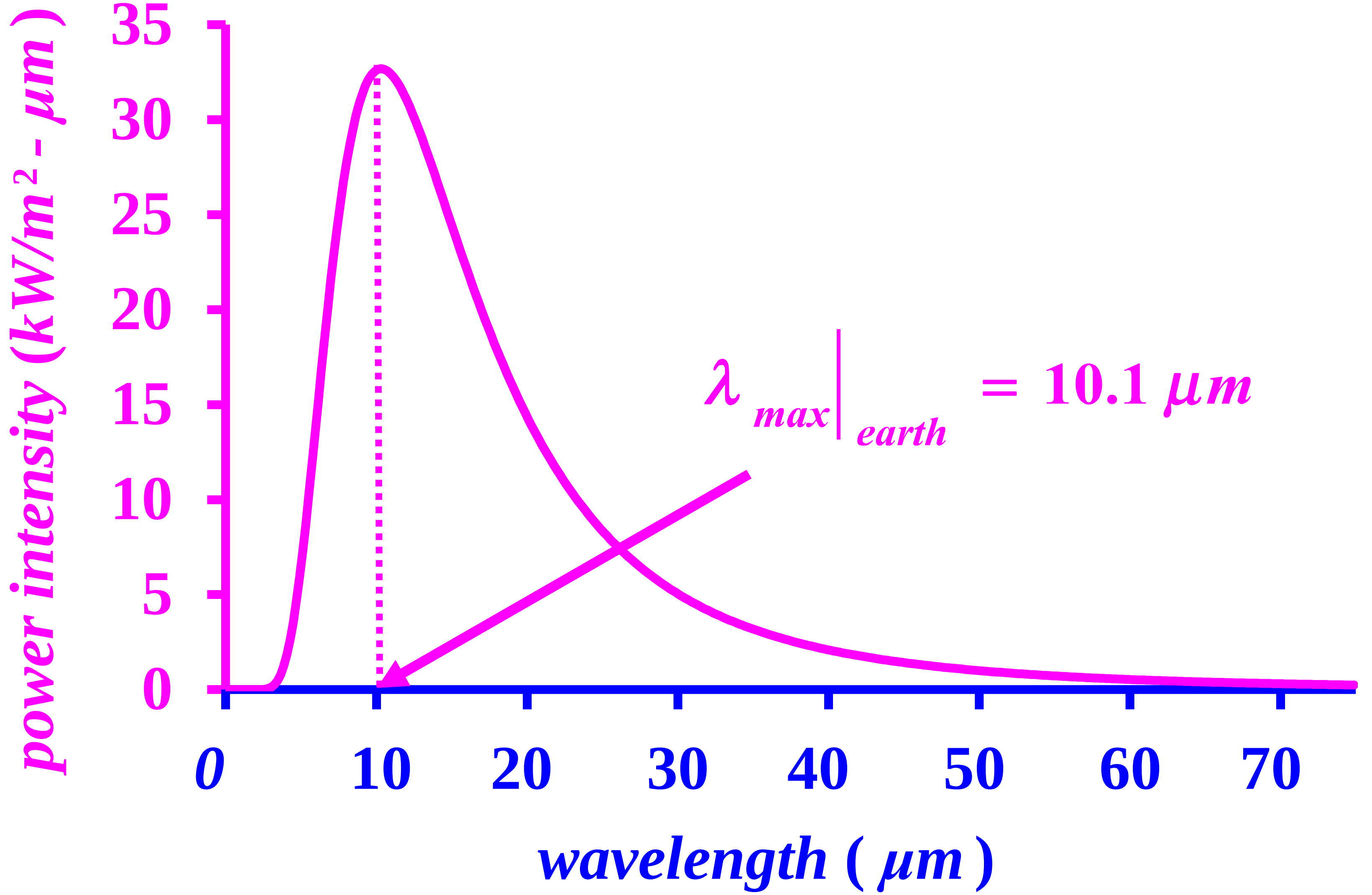
$$= (5.67 \times 10^{-8})(5.1 \times 10^{14})(15 + 273)$$

$$= 2 \times 10^{17}\ W$$

- ❑ The wavelength at which the maximum power is emitted is given by *Wien's displacement rule*

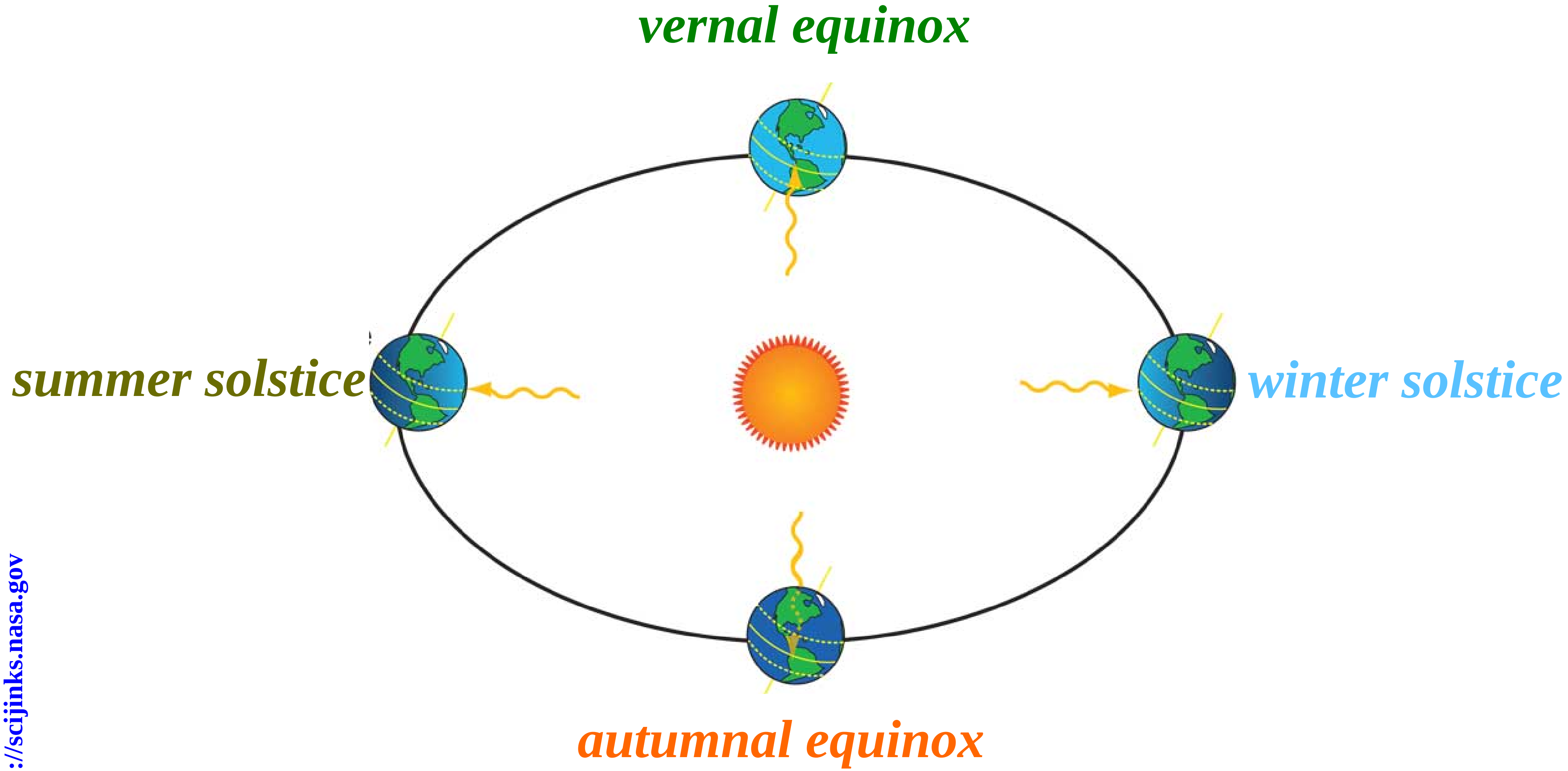
$$\lambda_{max}\Big|_{earth} = \frac{2,898}{288} = 10.1\ \mu m$$

THE SPECTRAL EMISSIVE POWER INTENSITY OF A 288 - *K* BLACKBODY



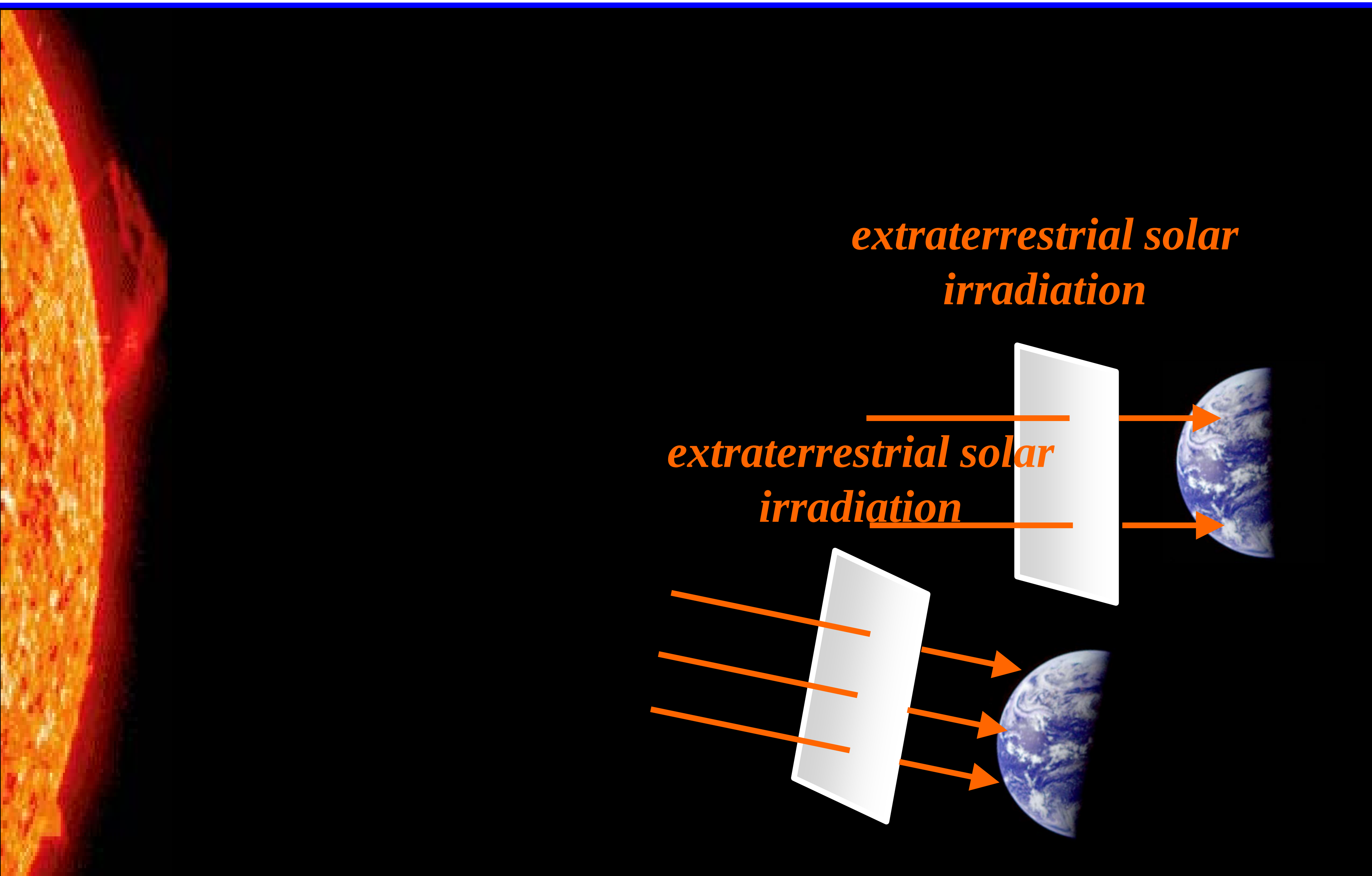
Source : http://www.ioccg.org/groups/mueller.html

EARTH'S ORBIT OVER ITS YEARLY REVOLUTION AROUND THE SUN



Source : http://scijinks.nasa.gov

EXTRATERRESTRIAL SOLAR IRRADIATION



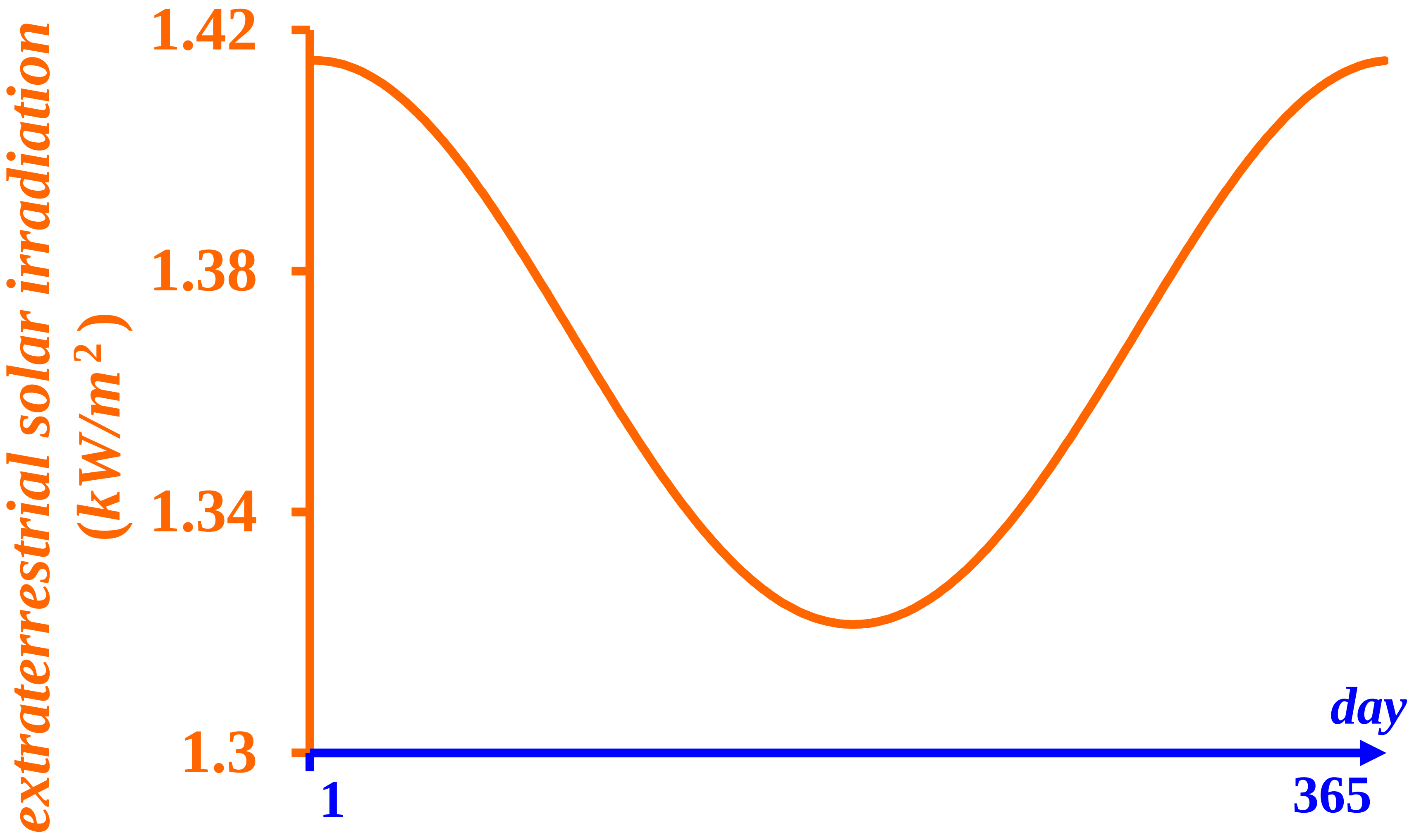
extraterrestrial solar irradiation

extraterrestrial solar irradiation

EXTRATERRESTRIAL SOLAR IRRADIATION OVER A YEAR

- In the analysis of solar issues, we use *solar time*, which is based on the sun's position with respect to the earth, instead of *clock* or *civil time*
- Extraterrestrial solar irradiation depends on the distance between the earth and the sun and therefore is a function of the day of the year

EXTRATERRESTRIAL SOLAR IRRADIATION OVER A YEAR



Source: http://solardat.uoregon.edu/SolarRadiationBasics.html/

EXTRATERRESTRIAL SOLAR IRRADIATION OVER A YEAR

- The extraterrestrial solar irradiation change during a day is negligibly small and so we assume that its value is constant as the earth rotates each day
- We use the approximation for $i_0\big|_d$ given by:

$$i_0\big|_d = 1,367\left[1 + 0.034\cos\left(2\pi\frac{d}{365}\right)\right] \qquad d = 1,2, \ldots, 365/366$$

W/m^2

EXTRATERRESTRIAL SOLAR IRRADIATION

- ❑ We consider the quantification of extraterrestrial solar irradiation on January 1: $d = 1$

$$i_0 \Big|_1 = 1,367 \left[1 + 0.034 \cos \left(2\pi \frac{1}{365} \right) \right] = 1,413 \frac{W}{m^2}$$

- ❑ Now, for August 1, $d = 213$ and the extraterrestrial solar irradiation is

$$i_0 \Big|_{213} = 1,367 \left[1 + 0.034 \cos \left(2\pi \frac{213}{365} \right) \right] = 1,326 \frac{W}{m^2}$$

EXTRATERRESTRIAL SOLAR IRRADIATION

- ❑ We observe that the extraterrestrial solar irradiation is higher in a cold winter day than in a hot summer day in the Northern hemisphere
- ❑ This phenomenon results from the fact that the sunlight enters into the atmosphere with different incident angles; these angles have major repercussions on the fraction of extraterrestrial

EXTRATERRESTRIAL SOLAR IRRADIATION

solar irradiation received on the earth's surface at different times of the year

- ❑ **As such, at a specified geographic location, we need to determine the *solar position in the sky* for the evaluation of the effective amount of solar irradiation at that location**

THE SOLAR POSITION IN THE SKY

The *solar position in the sky* varies as a function of:

○ the geographic location of interest;

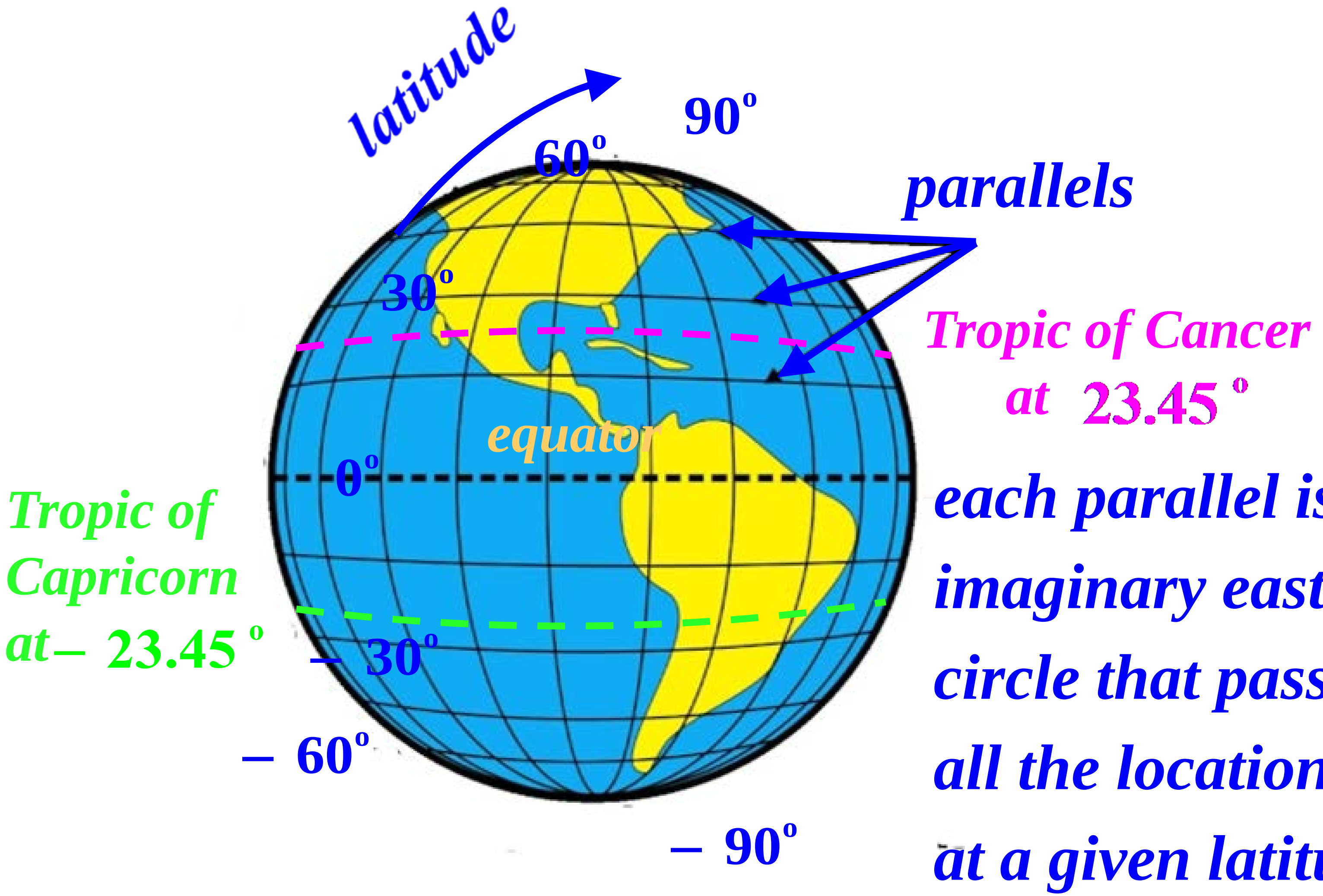
○ the time of day due to the earth's rotation around its tilted axis; and,

○ the day of the year that the earth is on its orbital revolution around the sun

LATITUDE AND LONGITUDE

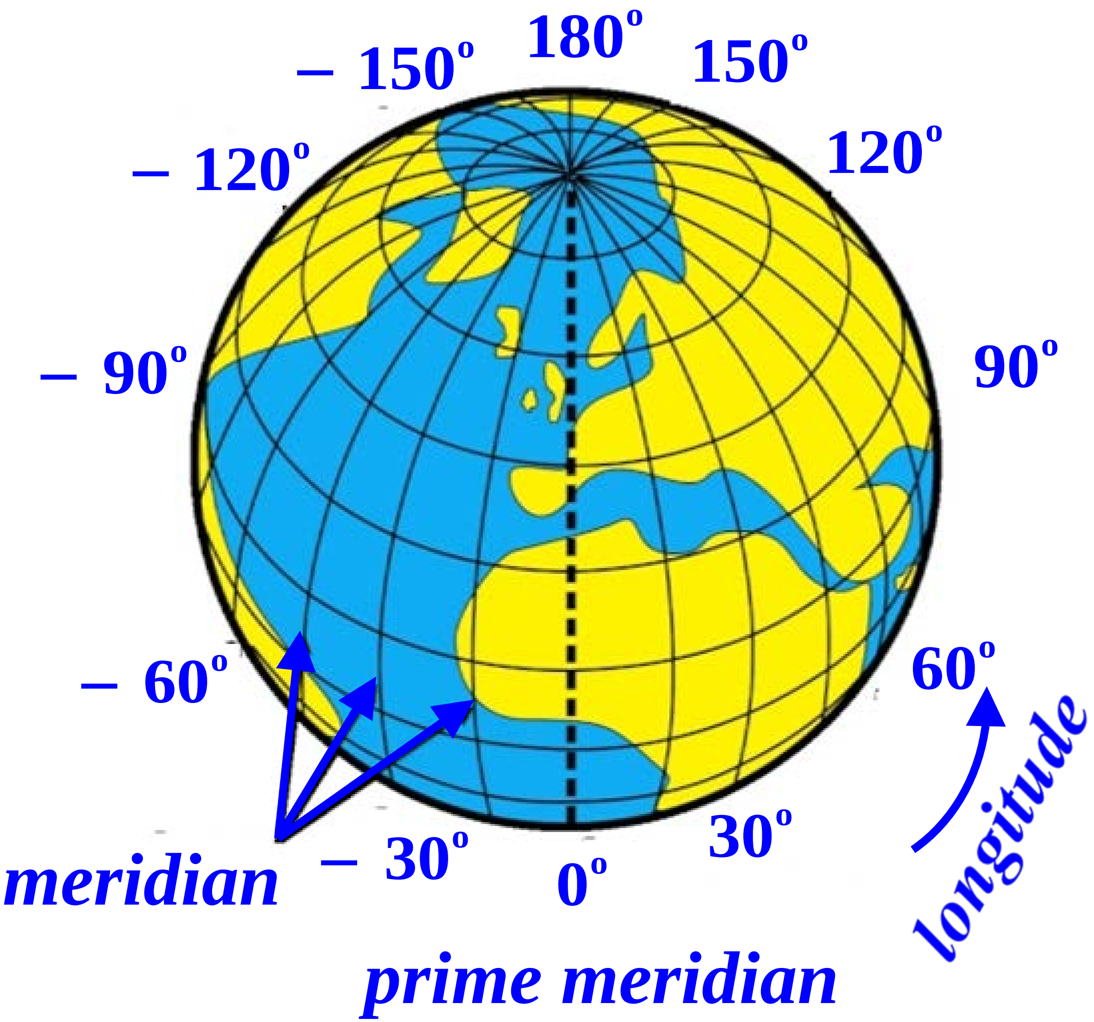
- A geographic location on earth is specified fully by the local *latitude* and *longitude*
- The *latitude* and *longitude* pair of geographic coordinates specify the North-South and the East-West positions of a location on the earth's surface; the coordinates are expressed in *degrees* or *radians*

LATITUDE AND LONGITUDE



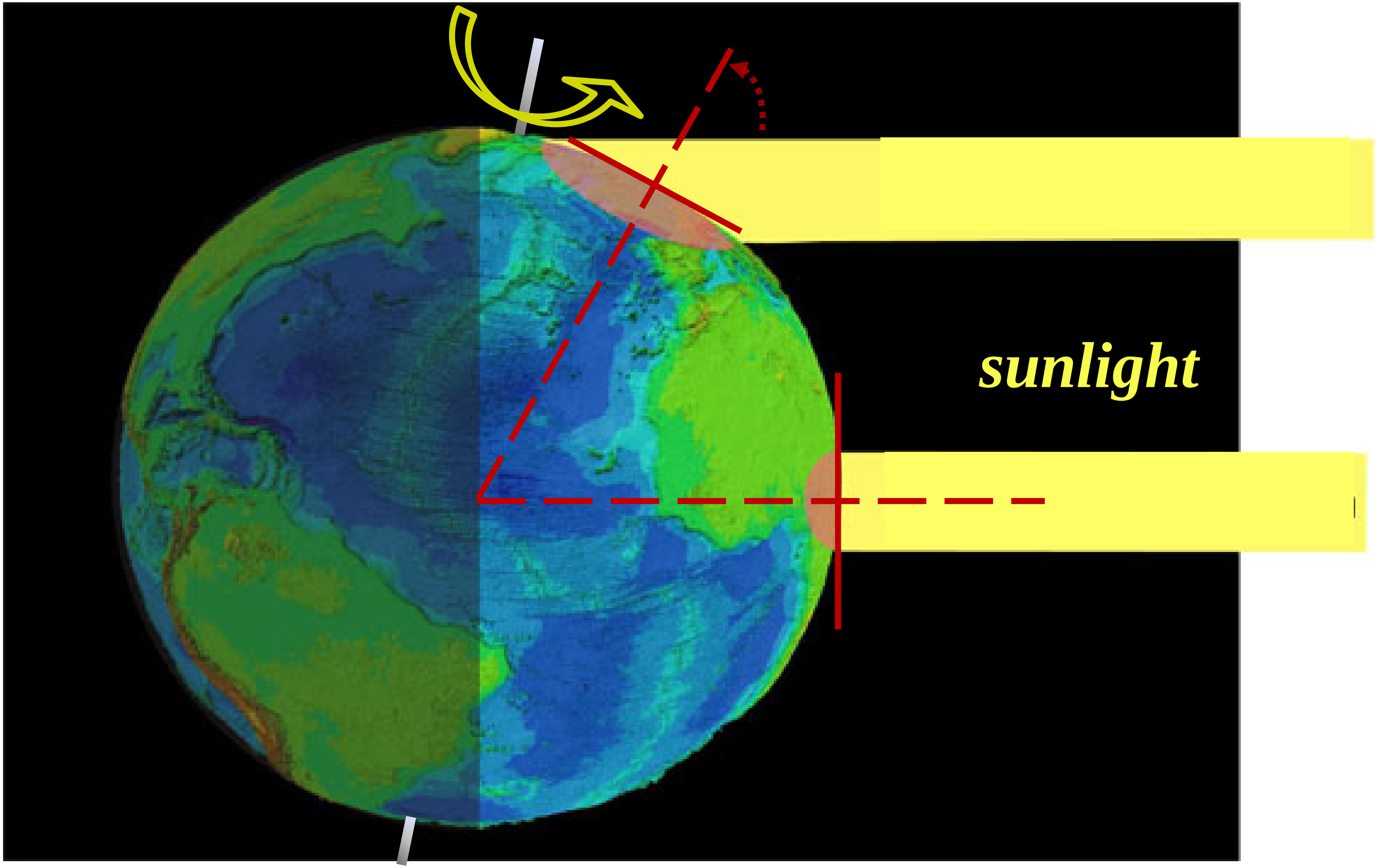
each parallel is an imaginary east-west circle that passes through all the locations that are at a given latitude

LATITUDE AND LONGITUDE

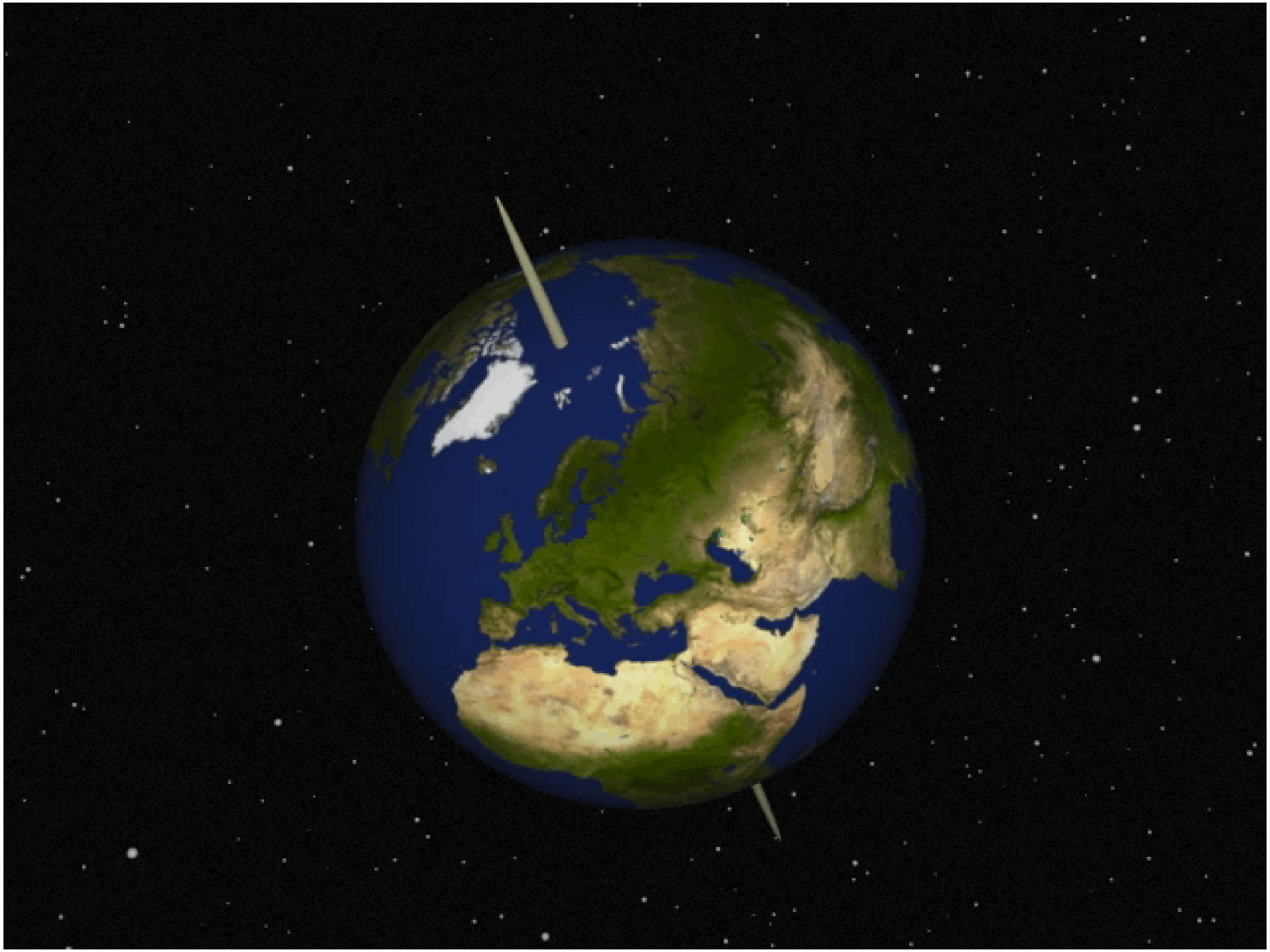


a meridian is an imaginary arc on the earth's surface between the North and South poles

THE SOLAR IRRADIATION VARIES BY THE GEOGRAPHIC LOCATION



EARTH'S ROTATION

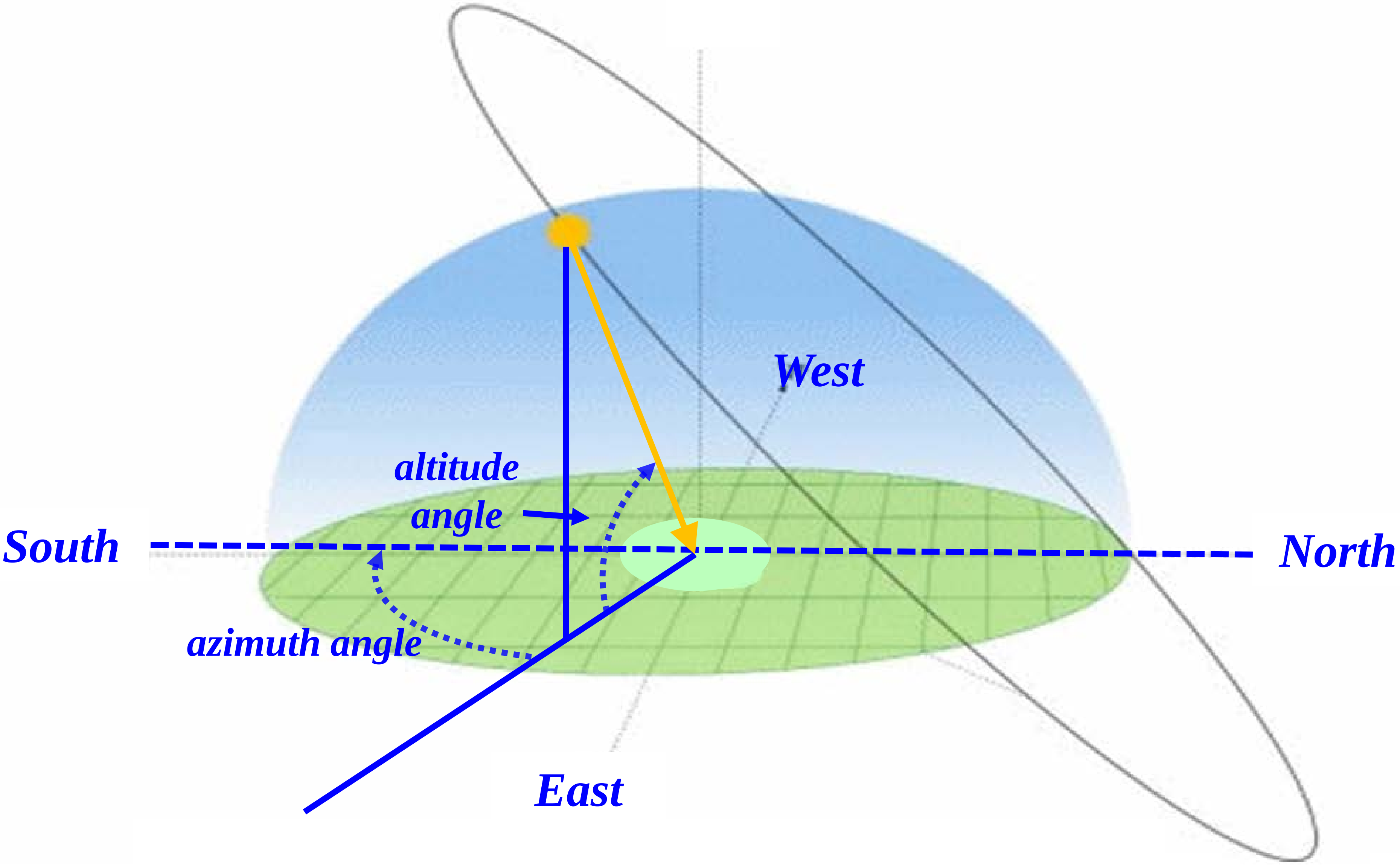


http://upload.wikimedia.org/wikipedia/commons/d/d7/Earth%27s_Axis.gif

EARTH'S ROTATION

- ☐ Although the sun's position is fixed in the space, earth's rotation around its tilted axis results in the "movement" of sun from east to west during each day's sunrise – sunset period
- ☐ The "movement" of the sun's position in the sky causes the variations in the solar irradiation at the specified location on the earth's surface

THE SOLAR IRRADIATION VARIES BY THE TIME OF A DAY



THE SOLAR POSITION IN THE SKY AT ANY TIME OF THE DAY

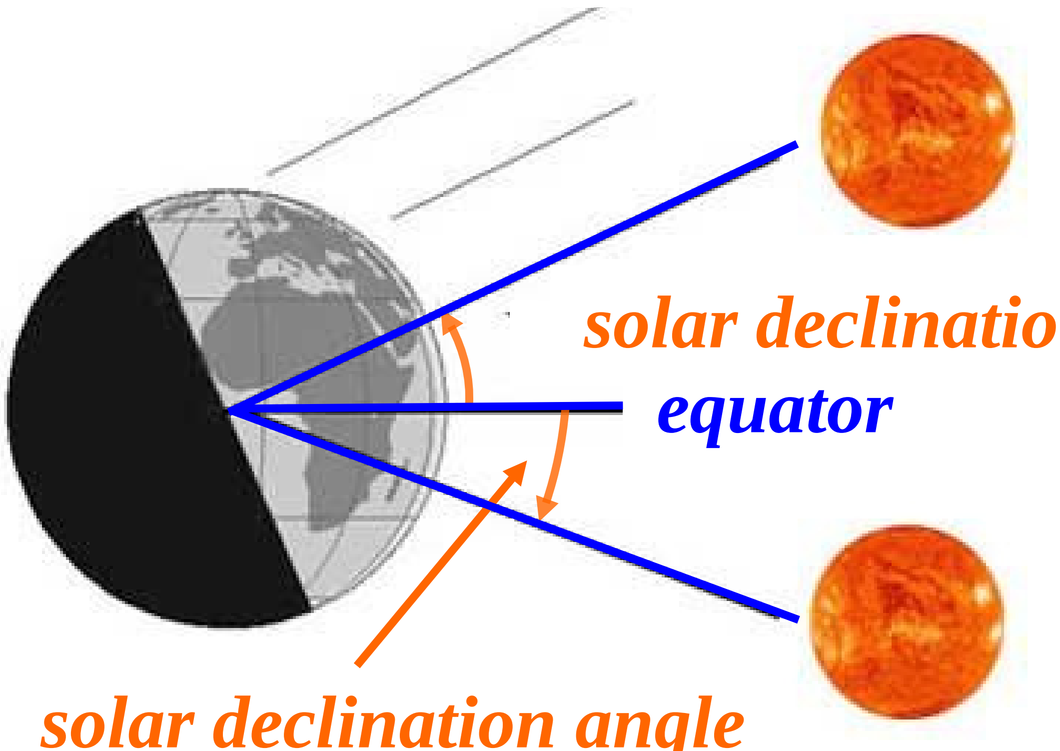
- **The solar position in the sky at any time of the day is expressed in terms of the *altitude angle* and the *solar azimuth angle***
- **The *altitude angle* is defined as the angle between the sun and the local horizon, which depends on the location's *latitude*, *solar declination angle* and *solar hour angle***

SOLAR DECLINATION ANGLE

- The *solar declination angle* refers to the angle between the plane of the equator and an imaginary line from the center of the sun to the center of the earth
- The change of *solar declination angle* during a day is sufficiently small and so we assume it to remain constant and represent it as a function of d by $\delta\big|_{d}$

SOLAR DECLINATION ANGLE

$$\delta\big|_{d} = 0.41\ \sin\left[\frac{2\pi}{365}(d-81)\right]\ radians$$



Source: https://encrypted-tbn3.gstatic.com/images?q=tbn:ANd9/

SOLAR HOUR ANGLE

- ❑ ***Solar noon*** **is the time at which the solar position in the sky is vertically over the local meridian, i.e., the line of longitude; in other words, the sun is due South (North) of the location in the Northern (Southern) Hemisphere**
- ❑ ***Solar hour angle*** $\theta(h)$ **refers to the angular rotation in radians the earth must go through to reach the solar noon;** ***h*** **is positive before the solar noon –** ***ante meridiem*** **– and negative after noon –** ***post meridiem***

SOLAR HOUR ANGLE

- ☐ We consider the earth to rotate at $\frac{2\pi}{24}$ per hour,

$$\theta(h) = \frac{\pi}{12} h \;\; radians$$

- ☐ At 11 *a.m.* in solar time

$$\theta(1) = \frac{\pi}{12}$$

and at 2 *p.m.* in solar time

$$\theta(-2) = -\frac{\pi}{6}$$

ALTITUDE ANGLE

Then, the relation of *altitude angle* $\beta(h)\big|_d$ and the location's *latitude*, *solar declination angle* and *solar hour angle* is given by

$$\sin\left(\beta(h)\big|_d\right)$$

$$= \cos(\ell)\cos\left(\delta\big|_d\right)\cos(\theta(h)) + \sin(\ell)\sin\left(\delta\big|_d\right)$$

where ℓ is the local latitude

EXAMPLE: ALTITUDE ANGLE AT CHAMPAIGN

- ☐ **Champaign's latitude is 0.7 *radians***
- ☐ **For October $22 - d = 295$, the solar declination angle is computed to be**

$$\delta\,\big|_{295} \;=\; 0.41\, sin\left[\frac{2\pi}{365}\left(295 - 81\right)\right] = -\,0.21\, radians$$

- ☐ **At $1\ p.m.$ in solar time, the hour angle is:**

$$\theta(-1) \;=\; \frac{\pi}{12}\cdot(-1) = -\,\frac{\pi}{12}\ radians$$

EXAMPLE: ALTITUDE ANGLE AT CHAMPAIGN

- ☐ We compute the *altitude angle* at Champaign from

$$sin\left(\beta(-1)\Big|_{295}\right)$$

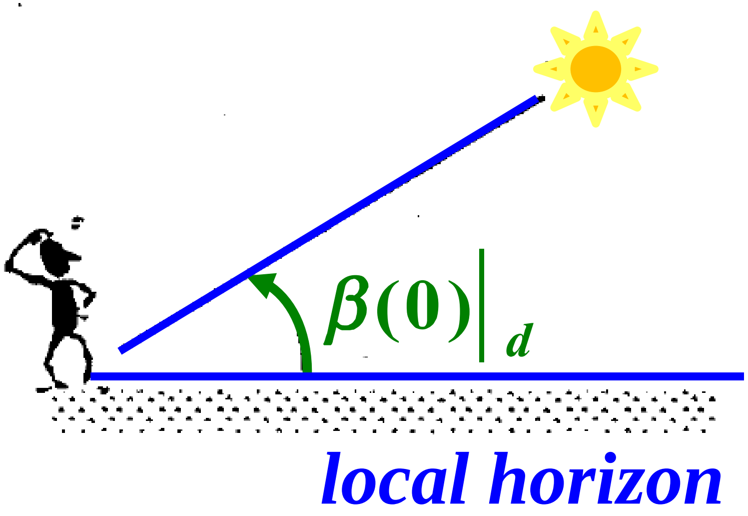
$$= cos(0.7)\,cos(-0.21)\,cos\left(-\frac{\pi}{12}\right) + sin(0.7)\,sin(-0.21)$$

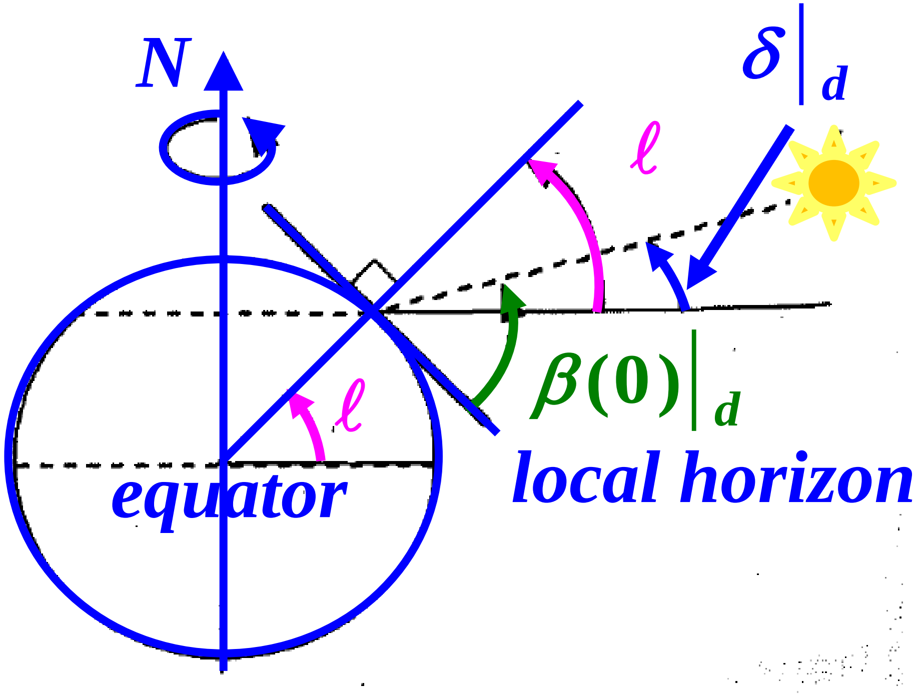
$$= 0.59$$

and so

$$\beta(-1)\Big|_{295} = sin^{-1}(0.59) = 0.623\ radians$$

SPECIAL CASE: THE ALTITUDE ANGLE AT SOLAR NOON





SPECIAL CASE: ALTITUDE ANGLE AT SOLAR NOON

- ☐ The *altitude angle at solar noon* of day d satisfies

$$sin\left(\beta(0)\Big|_{d}\right)$$

$$= cos(\ell)\, cos\left(\delta\Big|_{d}\right) cos\left(\theta(0)\right) + sin(\ell)\, sin\left(\delta\Big|_{d}\right)$$

- ☐ However, a more natural expression for $\beta(0)\Big|_{d}$ is obtained via the geometric relation

$$\beta(0)\Big|_{d} = \frac{\pi}{2} - \ell + \delta\Big|_{d} \quad radians$$

EXAMPLE: ALTITUDE ANGLE AT SOLAR NOON

- We determine the altitude angle for Champaign

  with $\ell = 0.7\ radians$, at solar noon on March 1

- The solar declination is

$$\delta\Big|_{60} = 0.41\ sin\left[\frac{2\pi}{365}\left(60 - 81\right)\right] = -\ 0.15\ radians$$

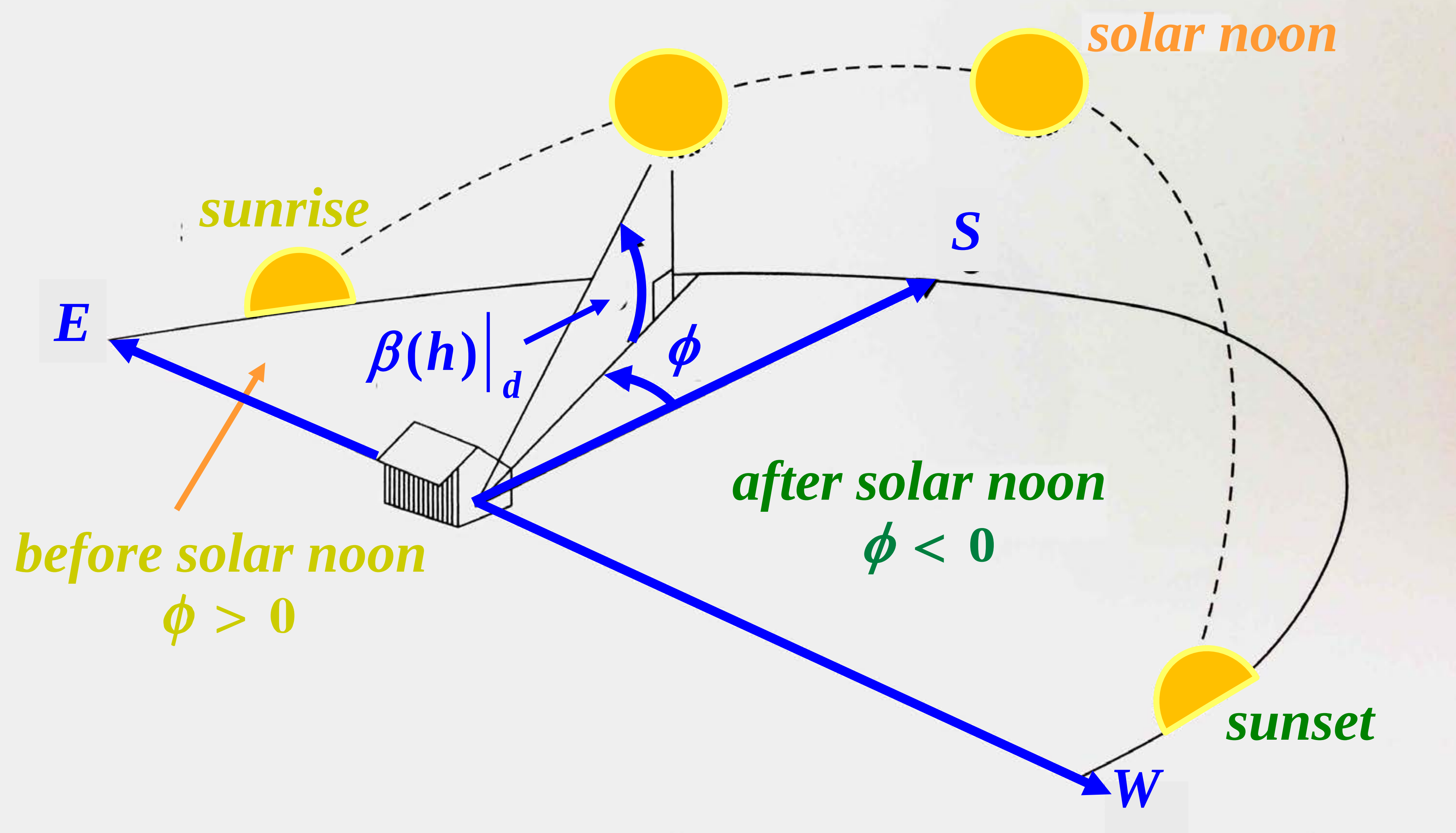
- The altitude angle at solar noon is

$$\beta(0)\Big|_{60} = \frac{\pi}{2} - \ell + \delta\Big|_{d} = 0.72\ radians$$

THE SOLAR AZIMUTH ANGLE

- *The solar azimuth angle* ϕ is defined as the angle between a due South line in the Northern Hemisphere and the projection of the line of sight to the sun on the earth surface
- We use the convention that ϕ is positive when the sun is in the East – before solar noon – and negative when the sun is in the West – after noon

THE SOLAR AZIMUTH ANGLE



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THE SOLAR AZIMUTH ANGLE

- ❑ **The equation for the *solar azimuth angle* $\phi(h)\big|_d$ is determined from the relationship**

$$sin\left(\phi(h)\big|_d\right) = \frac{cos\left(\delta\big|_d\right)\, sin(\theta(h))}{cos\left(\beta(h)\big|_d\right)}$$

- ❑ **Since the sinusoidal function is given to ambiguity as $sin\, x = sin\,(\pi - x)$, we need to**

THE SOLAR AZIMUTH ANGLE

determine whether the azimuth angle is greater or

less than $\frac{\pi}{2}$:

if $\quad cos\big(\theta\,(h)\big) \geq \frac{tan\,\big(\delta\big|_{d}\big)}{tan\,(\ell)}$ then $\quad \Big|\,\phi\,(h)\big|_{d}\Big| \leq \frac{\pi}{2}$

else $\quad \Big|\,\phi\,(h)\big|_{d}\Big| > \frac{\pi}{2}$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- ❑ **Determine the *altitude* and the *solar azimuth* angles at 3 *p.m.* in Champaign with latitude $\ell = 0.7$ *radians* at the summer solstice – $d = 172$**
- ❑ **The solar declination is**

$$\delta\Big|_{172} = 0.41 \ \textit{radians}$$

- ❑ **The hour angle at 3 *p.m.* is**

$$\theta(-3) = -\frac{\pi}{4}$$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- ☐ Then we compute the altitude angle:

$$\sin\left(\beta(-\ 3)\Big|_{172}\right)$$

$$= \cos(0.7)\cos\left(0.41\right)\cos\left(-\frac{\pi}{4}\right) + \sin(0.7)\sin\left(0.41\right)$$

$$= 0.75$$

- ☐ Then

$$\beta(-\ 3)\Big|_{172} = 0.85 \quad radians$$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- ☐ The sine of the azimuth angle is obtained from

$$sin\left(\phi(-3)\Big|_{172} \right) = \frac{cos\left(0.41\right)\ sin\left(-\frac{\pi}{4} \right)}{cos\left(0.85\right)} = -0.9848$$

- ☐ Two possible values for the azimuth angle are

$$\phi(-3)\Big|_{172} = sin^{-1}(-\ 0.9848) = -\ 1.4\ radians$$

or

$$\phi(-3)\Big|_{172} = \pi -\ sin^{-1}(-\ 0.9848) = 4.54\ radians$$

EXAMPLE: WHERE IS THE SUN IN THE SKY

- ☐ Since

$$\cos\left(\theta\,(-3)\right) = 0.707 \quad \text{and} \quad \frac{\tan\left(\delta\big|_{172}\right)}{\tan(\ell)} = 0.515$$

- ☐ Then we can determine

$$\cos\left(\theta\,(-3)\right) > \frac{\tan\left(\delta\big|_{172}\right)}{\tan(\ell)}$$

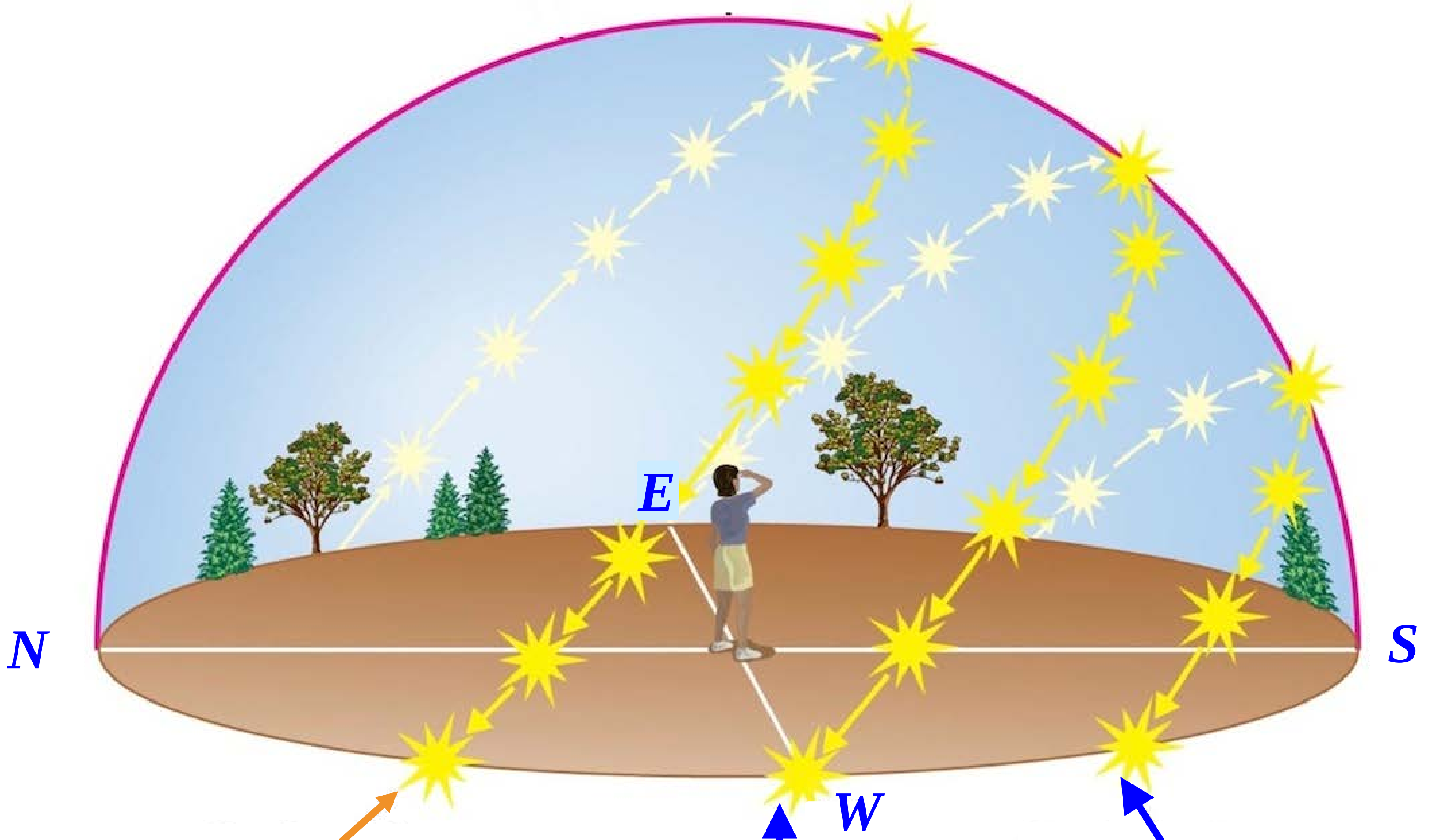
- ☐ Thus

$$\phi\,(-3)\big|_{172} = -\ 1.4\ radians$$

IMPORTANCE OF THE ANALYSIS ON SUN'S POSITION IN THE SKY

- We are now equipped to determine the sun's position in the sky at any time and at any location
- To effectively design and analyze solar plants, sun's position in the sky analysis has some highly significant applications, including to
  - build sun path diagram and do shading analysis
  - determine sunrise and sunset times
  - evaluate a solar panel's optimal *position*

SUN PATH



sun's path at the summer solstice

sun's path at an equinox

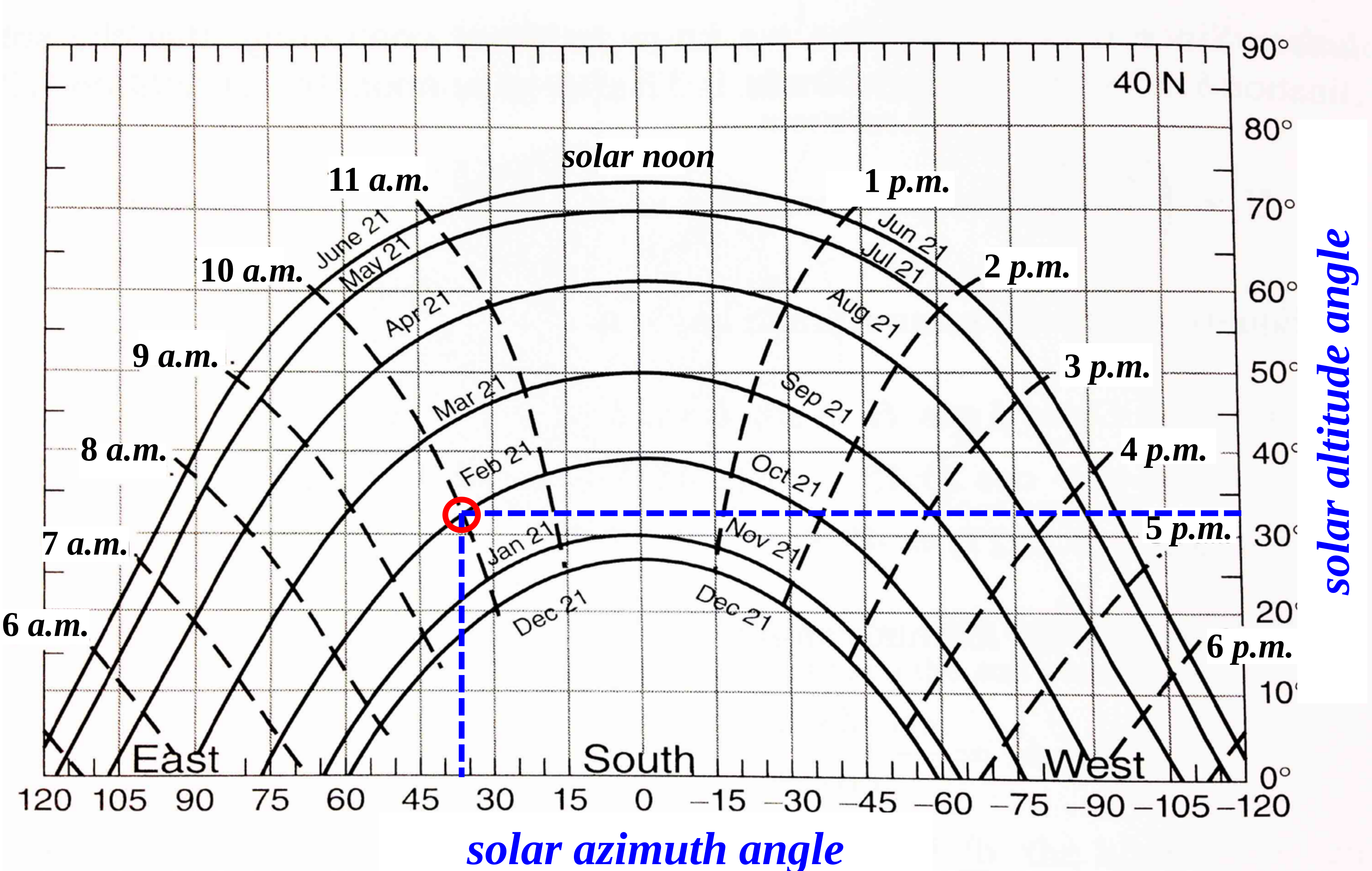
sun's path on winter solstice

SUN PATH DIAGRAM

- ☐ The *sun path diagram* is a chart used to illustrate the continuous changes of sun’s location in the sky at a specified location

- ☐ The sun’s position in the sky is found for any *hour* of the specified day *d* of the year by reading the *azimuth* and *altitude angles* in the *sun path diagram* corresponding to that *hour*

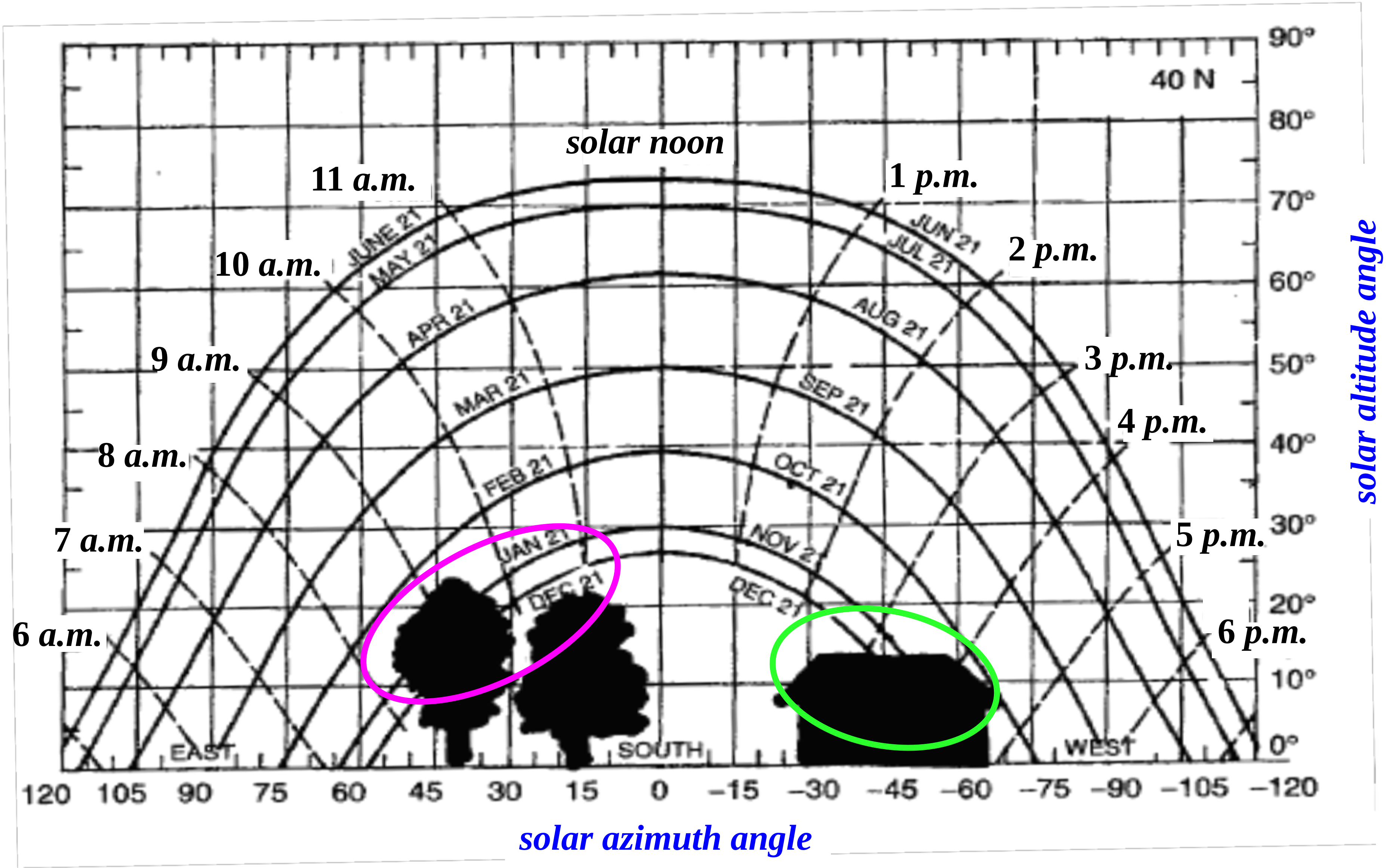
SUN PATH DIAGRAM FOR 40° *N*



SUN PATH DIAGRAM FOR SHADING ANALYSIS

- **In addition to the usefulness of *sun path diagrams* to help us find the sun’s position in the sky, they also have strong practical application in shading analysis at a site – an important issue in *PV* design due to the strong shadow sensitivity of *PV* output**
- **Modification of the sun path diagram for shading analysis requires a determination of the *azimuth* and *altitude angles* of the obstructions**

EXAMPLE: SUN PATH DIAGRAM FOR SHADING ANALYSIS



IMPORTANCE OF SHADING ANALYSIS

SHADING ANALYSIS USING SHADOW DIAGRAM

- In the set-up of a solar field, it is important to design the arrays so that the solar panels do not shade each other

- In addition to the application of sun path diagrams, there are other graphical and analytic approaches for shading analysis; such topics are outside the scope of the course

SUNRISE AND SUNSET

- An important issue is the determination of the sunrise/sunset times since solar energy is only collected during the sunrise to sunset hours
- We estimate the sunrise/sunset time from the equation used to compute the solar altitude angle, which is zero at sunrise and sunset

SUNRISE AND SUNSET

$$sin\left(\beta(h) \Big|_{d} \right) = 0$$

- ☐ The relationship for the solar angle results in:

$$cos\left(\theta(h)\right) = - \frac{sin(\ell)\, sin\left(\delta \Big|_{d} \right)}{cos(\ell)\, cos\left(\delta \Big|_{d} \right)} = - \; tan(\ell)\, tan\left(\delta \Big|_{d} \right)$$

- ☐ Now we can determine the sunrise solar hour

angle $\kappa_{+} \Big|_{d}$ and the sunset hour angle $\kappa_{-} \Big|_{d}$ to be:

SUNRISE AND SUNSET

- The corresponding sunrise and sunset angles are

$$\kappa_{+}\Big|_{d} = \cos^{-1}\Big(- \tan(\ell)\tan\big(\delta\big|_{d}\big)\Big)$$

$$\kappa_{-}\Big|_{d} = -\cos^{-1}\Big(- \tan(\ell)\tan\big(\delta\big|_{d}\big)\Big)$$

so that the solar times for sunrise/sunset are at

$$12:00 - \frac{\kappa_{+}\big|_{d}}{\pi/12} \quad \text{and} \quad 12:00 - \frac{\kappa_{-}\big|_{d}}{\pi/12}$$

SUNRISE TIME IN CHAMPAIGN

- Champaign is located at $\ell = 0.7 \ radians$
- On October 22, the solar declination angle is – 0.21 *radians* and the sunrise solar hour angle is :

$$\kappa_{+}\Big|_{295} = cos^{-1}\Big(- tan(0.7)\, tan\big(- 0.21 \big)\Big) = 1.39\, radians$$

- The sunrise expressed in solar time is at

$$12:00 - \frac{1.39}{\pi / 12} = 6:27\, a.m.$$

SOLAR TIME AND CIVIL TIME

- ☐ **So far, we used exclusively *solar time* measured with reference to solar noon in all our analysis of insolation and its impacts**
- ☐ **However, in our daily life we typically use *civil* or *clock time*, which measures the time to align with the earth's daily rotation over exactly 24 *hours***

SOLAR TIME AND CIVIL TIME

- ❑ **The difference at a specified location on the earth surface between the *solar time* and the *civil time* arises from the earth's uneven movement along its orbit of the annual rotation around the sun and the deviation of the local time meridian from the location longitude**

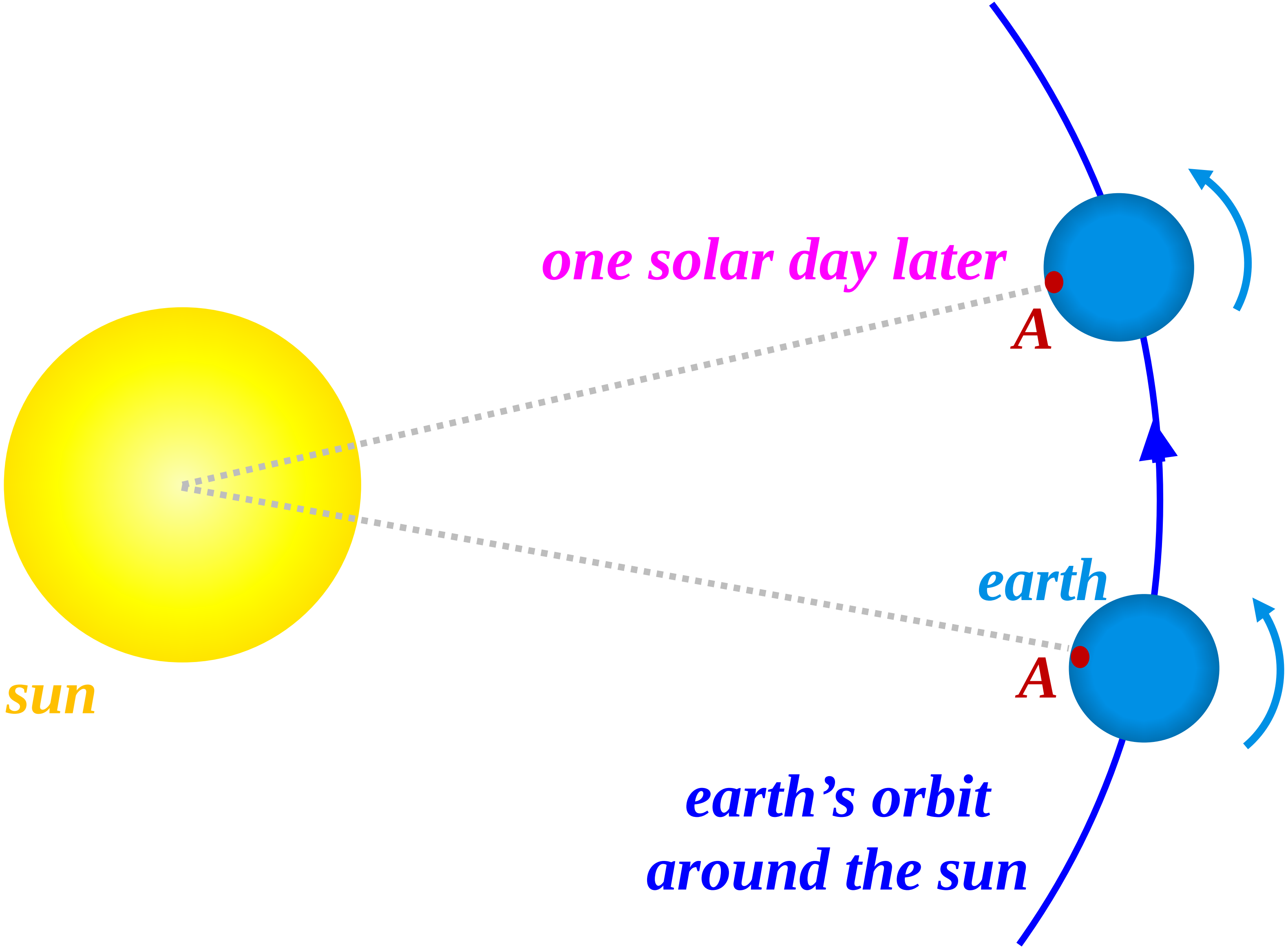
- ❑ **As such, two distinct adjustments must be made in order to convert between *solar time* and *civil time***

SOLAR DAY AND 24-*HOUR* DAY

- We examine the difference between a *solar day* and the corresponding 24-*hour* day

- A *solar day* is defined as the time elapsed between two successive solar noons

HOW LONG IS A SOLAR DAY



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SOLAR DAY

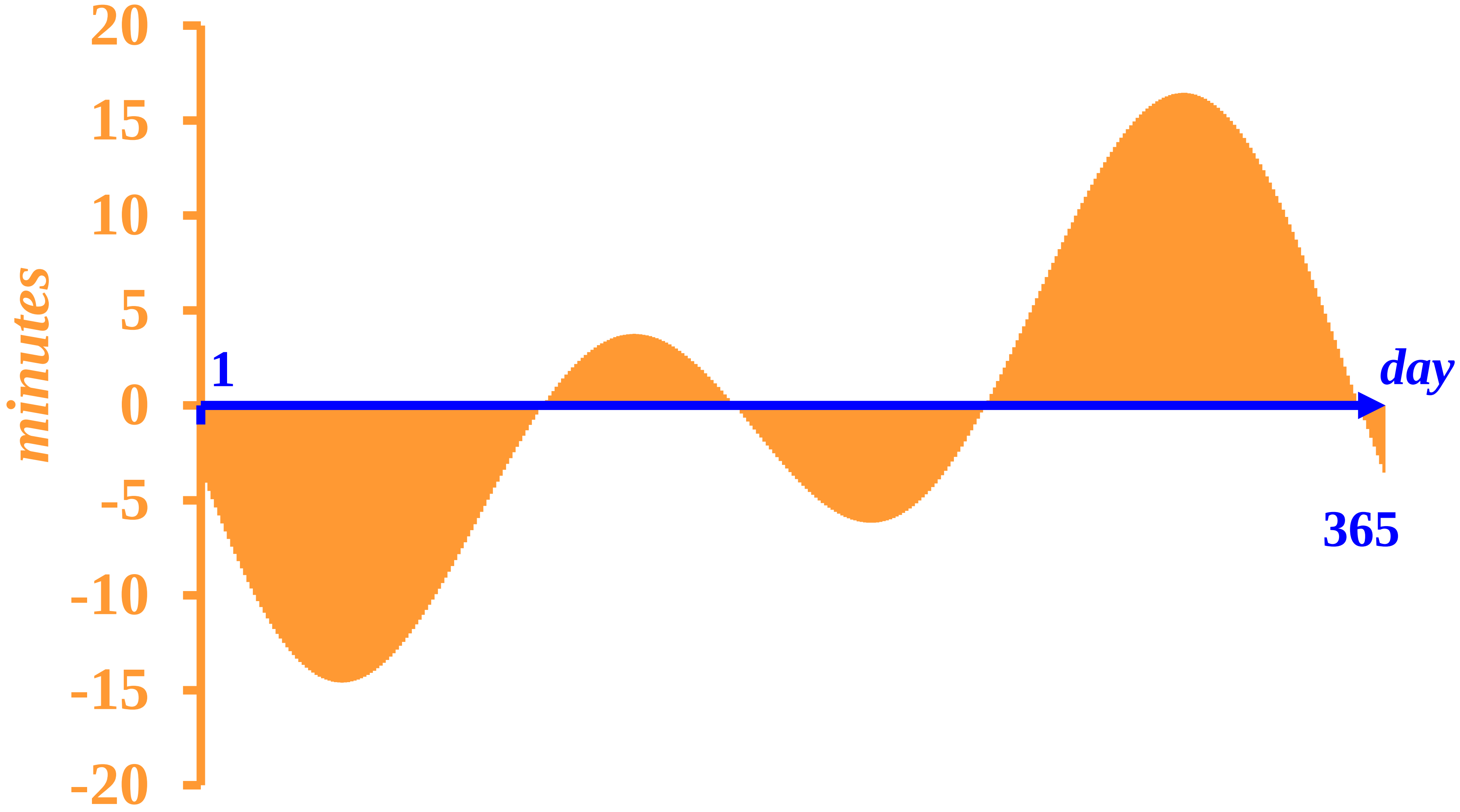
- ☐ The earth's elliptical orbit in its revolution around the sun results in varying durations of a solar day
- ☐ The difference between a solar day and a 24-*h* day is given by the deviation $e|_d$ in *minutes*

$$e|_d = 9.87 \, sin\left(2(b|_d)\right) - 7.53 \, cos(b|_d) - 1.5 \, sin\,(b|_d),$$

where,

$$b|_d = \frac{2\pi}{364}\left(d - 81\right) \;\; radians$$

DIFFERENCE BETWEEN A SOLAR AND A 24-*HOUR* DAY OVER A YEAR



LOCAL TIME MERIDIAN AND LOCAL LONGITUDE

- There are 24 time zones to cover the earth, each with its own time meridian with 15° longitude gap between the time meridians of two adjacent zones

- The second adjustment deals with the longitude correction for the fact that the clock time at any location within each time zone is defined by its local time meridian which differs from the time

LOCAL TIME MERIDIAN AND LOCAL LONGITUDE

- For every degree of longitude difference, the solar time difference corresponds to

$$\frac{24\ hour \cdot 60\ m\ /\ hour}{360^{o}} = 4\ \frac{m}{degree\ longitude}$$

- The time adjustment due to the degree longitude difference between the specified location and the local time meridian is the product of 4 times the longitude difference expressed in *minutes*

LOCAL TIME MERIDIAN AND LOCAL LONGITUDE

- ❑ The sum of the adjustment $e|_d$ and the longitude correction results in:

$$solar\ time = clock\ time + e|_d + 4 \times \frac{180}{3.14} \times (local\ time\ meridian - local\ longitude)$$

- ❑ This relationship allows the conversion between *solar time* and *civil time* at any location on earth

EXAMPLE: SOLAR TIME AND CLOCK TIME

- ☐ **Find the clock time of *solar noon* in Springfield on July 1, the 182nd day of the year**
- ☐ **For *d* = 182, we have**

$$b\big|_{182} = \frac{2\pi}{364}\left(182 - 81\right) = 1.72\ radians$$

$$e\big|_{182} = 9.87\, sin\,(2\, x\, 1.72) - 7.53\, cos\,(1.72) - 1.5\, sin\,(1.72)$$

$$= -\ 3.51\ mins$$

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EXAMPLE: SOLAR TIME TO CLOCK TIME

- **For Springfield with longitude 1.55 *radians*, the clock time in the central time zone is:**

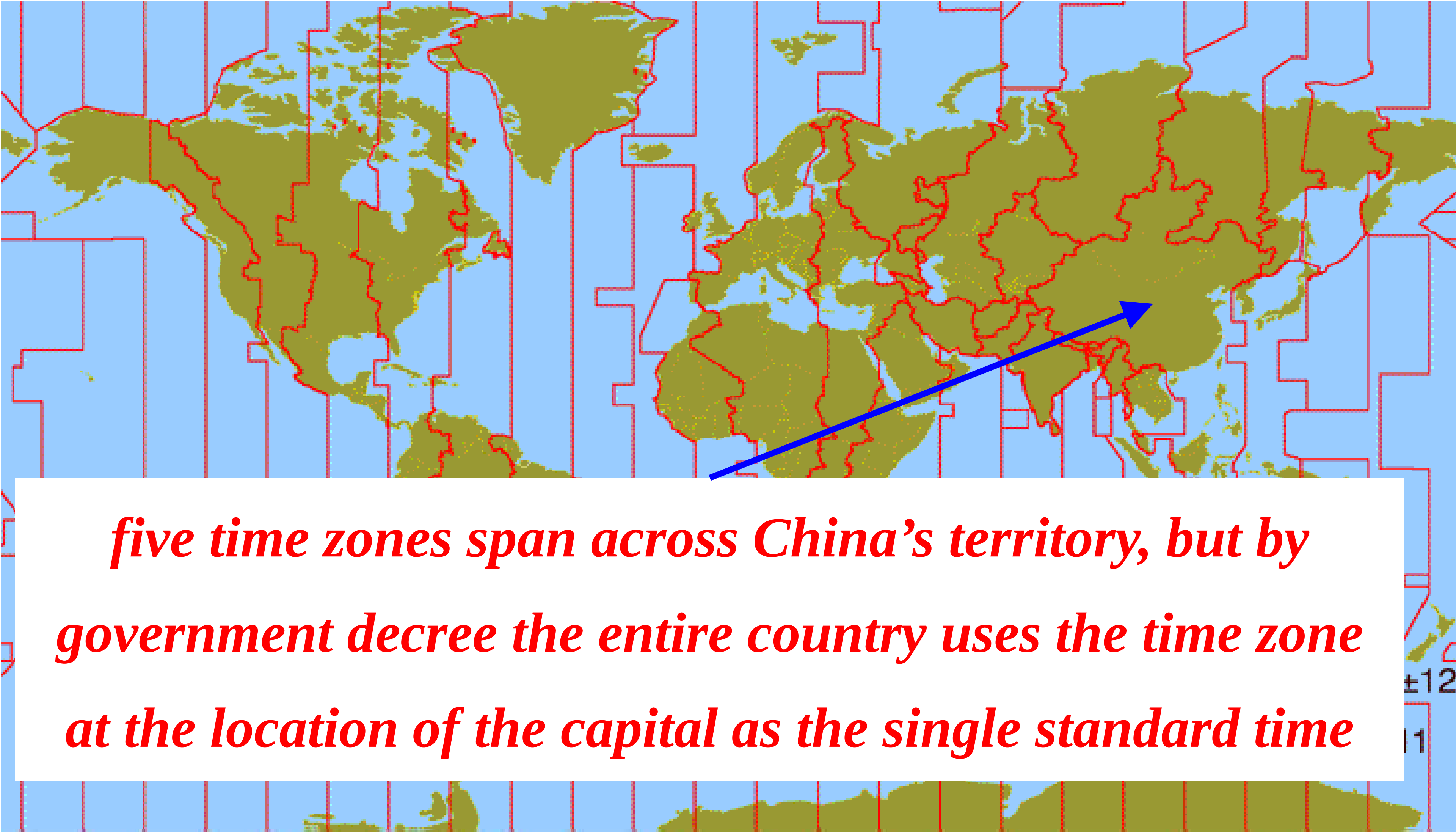
$$solar\ time\ -\ e\,|_{d}\ -\ 4 \times \frac{180}{3.14} \times$$

$$(local\ time\ meridian\ -\ local\ longitude)$$

$$=\ solar\ noon - (-3.51) - 57\,\big((-1.44) - (-1.55)\big)$$

$$=\ 11:38\ a.m.$$

WORLD TIME ZONE MAP



five time zones span across China's territory, but by government decree the entire country uses the time zone at the location of the capital as the single standard time

Source: http://www.physicalgeography.net/fundamentals/images/world_time2.gif

CONCLUSION

- ❑ With the conversion scheme between solar and clock times, the analysis on solar issues, e.g., the expression of sunrise/sunset in clock time, makes the results far more meaningful for use in everyday life
- ❑ Such a translation renders the results of the analysis to be much more concrete for applications